The Possible Textures in the Seesaw Realization of the Strong Scaling Ansatz and the Implications for Thermal Leptogenesis

Midori Obara *

Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918, Beijing 100049, China

February 2, 2008

Abstract

We classify the textures of the Dirac and the right-handed Majorana neutrino mass matrices, M_D and M_R, which can satisfy the so-called "Strong Scaling Ansatz" (SSA) within the framework of the seesaw mechanism $M_\nu = -M_D^T M_R^{-1} M_D$. We assume that the Dirac neutrino mass matrix has some texture zeros and examine which elements should be zero in order to satisfy the SSA, by taking into account all possible textures for M_R. We find that the resulting Dirac neutrino mass matrices have rank 2 as well as the rank of the effective neutrino mass matrix M_ν, or rank 1, depending only on the textures of M_R^{-1}. We also consider the three cases of the breaking of the SSA by introducing a complex breaking parameter in M_ν and show that it can generate the CP violation in the lepton sector as well as non-zero m_3 and U_{e3}. We furthermore discuss the implications of the thermal leptogenesis for the both cases which satisfy and break the SSA in the basis where M_R is diagonal.

1 Introduction

Since the discovery of neutrino oscillations by the Super-Kamiokande collaboration, solar, atmospheric, reactor and accelerator neutrino experiments (Super-Kamiokande [1], SNO [2], KamLAND [3], K2K [4] and MINOS [5]) have confirmed the evidence of neutrino oscillations. A global analysis of current experimental data yields [6]

$$\begin{aligned} 0.26 \leq &\ \sin^2\theta_{12} \leq 0.40 \ , \\ 0.34 \leq &\ \sin^2\theta_{23} \leq 0.67 \ , \\ &\ \sin^2\theta_{13} \leq 0.050 \ , \end{aligned} \tag{1.1}$$

*E-mail: midori@mail.ihep.ac.cn

and

$$
\begin{aligned}
7.1\times10^{-5}\ \mathrm{eV}^2 \leq &\quad \Delta m^2_{21} &\leq 8.3\times10^{-5}\ \mathrm{eV}^2\ ,\\
2.0\times10^{-3}\ \mathrm{eV}^2 \leq &\quad |\Delta m^2_{32}| &\leq 2.8\times10^{-5}\ \mathrm{eV}^2\ ,
\end{aligned} \tag{1.2}
$$

at the 3σ.

The structures of neutrino mass matrices have been studied in various models based on both continuous [7] and discrete flavor symmetries [8], and many attempts to connect the flavor symmetry approaches to the grand unified thories have been done [9]. However, these kind of approaches generally receive the corrections from the renormalization group effects. As a new approach independent of the renormalization group effects[1], R.N. Mohapatra and W. Rodejohann have recently proposed the strong scaling Ansatz (SSA) that the elements of the neutrino mass matrix $(M_\nu)_{\alpha\beta} \equiv m_{\alpha\beta}$ $(\alpha, \beta = e, \mu, \tau)$ satisfy the following scaling

$$
\frac{m_{e\mu}}{m_{e\tau}} = \frac{m_{\mu\mu}}{m_{\mu\tau}} = \frac{m_{\tau\mu}}{m_{\tau\tau}} \equiv c\,, \tag{1.3}
$$

in the basis where the charged lepton mass matrix is diagonal, and have shown that such a neutrino mass matrix

$$
M_\nu = \begin{pmatrix} A & B & B/c \\ B & D & D/c \\ B/c & D/c & D/c^2 \end{pmatrix} = U\mathrm{diag}(m_1, m_2, m_3)U^T\,, \tag{1.4}
$$

where U is the PMNS matrix, predicts the inverted hierarchy with $m_3 = 0$, vanishing U_{e3} and no CP violation [10], accommodating to the currecnt neutrino experimental data. Here we take the following parameterization for U [12]:

$$
U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} P\,, \tag{1.5}
$$

where $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$ and $P = \mathrm{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)})$ with α, β and δ being the Majorana and Dirac phases. By adjusting the value of the scaling parameter c, we can obtain the non-maximal atmospheric neutrino mixing angle which may be favored in future experiments. There are three possible cases of breaking the SSA [10]:

$$
\mathrm{A1}: \quad \frac{m_{e\mu}}{m_{e\tau}} = \frac{m_{\mu\mu}}{m_{\mu\tau}} = c\,, \quad \frac{m_{\tau\mu}}{m_{\tau\tau}} = c(1+\epsilon)\,, \tag{1.6}
$$

$$
\mathrm{A2}: \quad \frac{m_{e\mu}}{m_{e\tau}} = \frac{m_{\tau\mu}}{m_{\tau\tau}} = c\,, \quad \frac{m_{\mu\mu}}{m_{\mu\tau}} = c(1+\epsilon)\,, \tag{1.7}
$$

$$
\mathrm{A3}: \quad \frac{m_{\mu\mu}}{m_{\mu\tau}} = \frac{m_{\tau\mu}}{m_{\tau\tau}} = c\,, \quad \frac{m_{e\mu}}{m_{e\tau}} = c(1+\epsilon)\,, \tag{1.8}
$$

[1] It has been mentioned that the storong scaling may not be stable under radiative corrections in the MSSM for large value of $\tan\beta \sim 58 - 60$ in Ref. [11]

and it has been shown that non-zero m_3 and U_{e3} can be generated in these three cases [10]. In this paper we will show that both real and complex breaking parameter can generate the CP violation in the lepton sector as well as non-zero m_3 and U_{e3}.

This paper is organized as follows. In section 2, we classify the textures of the Dirac and the right-handed Majorana neutrino mass matrices, M_D and M_R, which can satisfy the SSA within the framework of the seesaw mechanism $M_\nu = -M_D^T M_R^{-1} M_D$, and show the conditions of elements in M_D for getting the SSA by taking into account all possible textures for M_R. In this section, we also consider the three cases of breaking the SSA by introducing a complex breaking parameter in M_ν and examine which cases of the breaking can be realized within the seesaw framework. In section 3, we briefly review the phenomenology of the SSA and examine the effects of the breaking of the SSA on m_3, U_{e3} and $J_{\rm CP}$ for the three cases A1, A2 and A3, semi-analytically. Numerical analyses for the original case of the SSA and the three cases A1, A2 and A3 will be done in section 4. In section 5, we discuss the implications of the thermal leptogenesis for the both cases which satisfy and break the SSA in the basis where M_R is diagonal. Section 6 is devoted to summary.

2 Classification

In this section, we classify the textures of M_D and M_R, which can satisfy the SSA within the framework of the seesaw mechanism $M_\nu = -M_D^T M_R^{-1} M_D$. In order to do that, we take the form of M_D as

$$M_D = \begin{pmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{pmatrix} , \qquad (2.1)$$

and find the conditions of elements in M_D for getting the SSA, by taking into account all possible textures for M_R. First, we mention the most general condition of the elements in M_D. If M_D is taken to be the following form,

$$M_D = \begin{pmatrix} a_1 & b_1 & b_1/c \\ a_2 & b_2 & b_2/c \\ a_3 & b_3 & b_3/c \end{pmatrix} , \qquad (2.2)$$

then M_ν satisfies the strong scaling, without depending on the textures for M_R [13]. Here we assume that M_D has some texture zeros and examine which elements should be zero in order to get the SSA. The Dirac neutrino mass matrices with texture zeros are attractive to relate the low energy CP violation in the lepton sector to the high energy CP violation necessary for the thermal leptogeneis [14, 15, 16, 17]. We have listed the conditions of elements in M_D for getting the SSA in Tables 1, 2, 3 and 4. It is obvious that the textures of M_R for the classes $\tilde{F}_1$–$\tilde{F}_7$ in Table 3 can be obtained by exchanging the generation indices of those for the classes F_1–F_7 in

Table 2. Similarly, the classes G_2 and G_3 can be obtained from the classes G_4 and G_5 in Table 4. From our classification, we have found that the resulting Dirac neutrino mass matrices have rank 2 as well as the rank of the effective neutrino mass matrix M_ν, or rank 1, depending only on the textures of M_R^{-1}; the same texture for M_R^{-1} leads to the same conditions for the elements in M_D.

We also consider the three cases of breaking the SSA by introducing a complex breaking parameter ϵ in M_ν. Here, all parameters are supposed to be complex, and after rephasing, we can redefine c, A, B and D as real and take the neutrino mass matrices for the three cases A1, A2 and A3 given in Eqs. (1.6), (1.7) and (1.8) as

$$\mathrm{A1}: \quad M_\nu = \begin{pmatrix} A & Be^{i\phi} & Be^{i\phi}/c \\ Be^{i\phi} & D(1+\epsilon) & D(1+\epsilon)/c \\ Be^{i\phi}/c & D(1+\epsilon)/c & D/c^2 \end{pmatrix}, \tag{2.3}$$

$$\mathrm{A2}: \quad M_\nu = \begin{pmatrix} A & Be^{i\phi} & Be^{i\phi}/c \\ Be^{i\phi} & D(1+\epsilon) & D/c \\ Be^{i\phi}/c & D/c & D/c^2 \end{pmatrix}, \tag{2.4}$$

$$\mathrm{A3}: \quad M_\nu = \begin{pmatrix} A & Be^{i\phi}(1+\epsilon) & Be^{i\phi}/c \\ Be^{i\phi}(1+\epsilon) & D & D/c \\ Be^{i\phi}/c & D/c & D/c^2 \end{pmatrix}, \tag{2.5}$$

where $\epsilon \equiv |\epsilon|e^{i\varphi}$ and $|\epsilon| \ll 1$. The conditions of elements in M_D for breaking the SSA have also been listed in Tables 1, 2, 3 and 4. From these tables, we have found that in the class E the cases A1 and A2 can be separately realized and the case A3 can only appear in combination with the two cases A1 or A2. In the other classes, only the case A3 can be separately realized and admixture of all three cases can also be possible.

Next, we will briefly review the phenomenology of the SSA and examine the effects of the breaking of the SSA on m_3, U_{e3} and J_{CP} for the three cases A1, A2 and A3, semi-analytically.

3 Neutrino masses and mixing angles in the SSA and the effect of the breaking of the SSA

Let us decompose the neutrino mass matrices for the cases A1, A2 and A3 as

$$M_\nu = M_\nu^{(0)} + M_\nu^{(1)}, \tag{3.1}$$

where

$$M_\nu^{(0)} = \begin{pmatrix} A & Be^{i\phi} & Be^{i\phi}/c \\ Be^{i\phi} & D & D/c \\ Be^{i\phi}/c & D/c & D/c^2 \end{pmatrix}, \tag{3.2}$$

Table 1: The texture of M_R and the conditions of elements in M_D for getting and breaking the SSA for the class E.

Class	M_R	Conditions for getting the SSA	Conditions for breaking the SSA
E	$\begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}$	(e1) $a_1 = b_1 = d_1 = 0$ and $b_2 = d_2 = 0$	$b_1 \neq 0$ (A2), $d_1 \neq 0$ (A1) $b_2 \neq 0$ (A2, A3), $d_2 \neq 0$ (A1, A3)
		(e2) $a_1 = b_1 = d_1 = 0$ and $b_3 = d_3 = 0$	$b_1 \neq 0$ (A2), $d_1 \neq 0$ (A1) $b_3 \neq 0$ (A2, A3), $d_3 \neq 0$ (A1, A3)
		(e3) $a_2 = b_2 = d_2 = 0$ and $b_1 = d_1 = 0$	$b_2 \neq 0$ (A2), $d_2 \neq 0$ (A1) $b_1 \neq 0$ (A2, A3), $d_1 \neq 0$ (A1, A3)
		(e4) $a_2 = b_2 = d_2 = 0$ and $b_3 = d_3 = 0$	$b_2 \neq 0$ (A2), $d_2 \neq 0$ (A1) $b_3 \neq 0$ (A2, A3), $d_3 \neq 0$ (A1, A3)
		(e5) $a_3 = b_3 = d_3 = 0$ and $b_1 = d_1 = 0$	$b_3 \neq 0$ (A2), $d_3 \neq 0$ (A1) $b_1 \neq 0$ (A2, A3), $d_1 \neq 0$ (A1, A3)
		(e6) $a_3 = b_3 = d_3 = 0$ and $b_2 = d_2 = 0$	$b_3 \neq 0$ (A2), $d_3 \neq 0$ (A1) $b_2 \neq 0$ (A2, A3), $d_2 \neq 0$ (A1, A3)

and

$$\mathrm{A1}: \quad M_\nu^{(1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & D|\epsilon|e^{i\varphi} & D|\epsilon|e^{i\varphi}/c \\ 0 & D|\epsilon|e^{i\varphi}/c & 0 \end{pmatrix}, \tag{3.3}$$

$$\mathrm{A2}: \quad M_\nu^{(1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & D|\epsilon|e^{i\varphi} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{3.4}$$

$$\mathrm{A3}: \quad M_\nu^{(1)} = \begin{pmatrix} 0 & B|\epsilon|e^{i(\phi+\varphi)} & 0 \\ B|\epsilon|e^{i(\phi+\varphi)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{3.5}$$

We diagonalize the mass matrix as $U_\nu^\dagger M_\nu M_\nu^\dagger U_\nu = \mathrm{diag}(m_1^2, m_2^2, m_3^2)$ by using the following decomposition:

$$M_\nu M_\nu^\dagger = M_\nu^{(0)} M_\nu^{(0)\dagger} + \delta\mathcal{M}\,, \tag{3.6}$$

where

$$\delta\mathcal{M} \equiv M_\nu^{(0)} M_\nu^{(1)\dagger} + M_\nu^{(1)\dagger} M_\nu^{(0)} + M_\nu^{(1)} M_\nu^{(1)\dagger}\,, \tag{3.7}$$

and then the unitary matrix for diagonalization of Eq. (3.6) is decomposed as

$$U_\nu \equiv U_\nu^{(0)} + U_\nu^{(1)}\,. \tag{3.8}$$

Table 2: The textures of M_R and the conditions of elements in M_D for getting and breaking the SSA for the classes F_1–F_7. Here, “All” means the admixture of all three cases A1, A2 and A3.

Class	M_R	Conditions for getting the SSA	Conditions for breaking the SSA
F_1	$\begin{pmatrix} 0 & s & 0 \\ s & 0 & 0 \\ 0 & 0 & x \end{pmatrix}$	(f1I) $a_1 = b_1 = d_1 = 0$ (f1II) $a_2 = b_2 = d_2 = 0$	$a_1 \neq 0$ (A3), $b_1 \neq 0$ (All), $d_1 \neq 0$ (All) $a_2 \neq 0$ (A3), $b_2 \neq 0$ (All), $d_2 \neq 0$ (All)
F_2	$\begin{pmatrix} 0 & s & 0 \\ s & t & 0 \\ 0 & 0 & x \end{pmatrix}$	(f2I) $a_1 = b_1 = d_1 = 0$ (f2II) $a_2 = b_2 = d_2 = 0$ and $b_3 = d_3 = 0$	$a_1 \neq 0$ (A3), $b_1 \neq 0$ (All), $d_1 \neq 0$ (All) $a_2 \neq 0$ (A3), $b_2 \neq 0$ (All), $d_2 \neq 0$ (All) $b_3 \neq 0$ (All), $d_3 \neq 0$ (All)
F_3	$\begin{pmatrix} 0 & s & 0 \\ s & t & u \\ 0 & u & x \end{pmatrix}$	(f3I) $a_1 = b_1 = d_1 = 0$ (f3II) $a_2 = b_2 = d_2 = 0$ and $a_3 = b_3 = d_3 = 0$	$a_1 \neq 0$ (A3), $b_1 \neq 0$ (All), $d_1 \neq 0$ (All) $a_2 \neq 0$ (A3), $b_2 \neq 0$ (All), $d_2 \neq 0$ (All) $a_3 \neq 0$ (A3), $b_3 \neq 0$ (All), $d_3 \neq 0$ (All)
F_4	$\begin{pmatrix} 0 & s & 0 \\ s & 0 & u \\ 0 & u & x \end{pmatrix}$	same as F_3	same as F_3
F_5	$\begin{pmatrix} 0 & s & z \\ s & 0 & u \\ z & u & x \end{pmatrix}$	(f5I) $a_1 = b_1 = d_1 = 0$ and $a_2 = b_2 = d_2 = 0$ (f5II) $a_2 = b_2 = d_2 = 0$ and $a_3 = b_3 = d_3 = 0$ (f5III) $a_1 = b_1 = d_1 = 0$ and $a_3 = b_3 = d_3 = 0$	$a_1 \neq 0$ (A3), $b_1 \neq 0$ (All), $d_1 \neq 0$ (All) $a_2 \neq 0$ (A3), $b_2 \neq 0$ (All), $d_2 \neq 0$ (All) $a_2 \neq 0$ (A3), $b_2 \neq 0$ (All), $d_2 \neq 0$ (All) $a_3 \neq 0$ (A3), $b_3 \neq 0$ (All), $d_3 \neq 0$ (All) $a_1 \neq 0$ (A3), $b_1 \neq 0$ (All), $d_1 \neq 0$ (All) $a_3 \neq 0$ (A3), $b_3 \neq 0$ (All), $d_3 \neq 0$ (All)
F_6	$\begin{pmatrix} 0 & s & z \\ s & t & u \\ z & u & x \end{pmatrix}$	same as F_5	same as F_5
F_7	$\begin{pmatrix} y & s & z \\ s & t & 0 \\ z & 0 & x \end{pmatrix}$	same as F_5	same as F_5

Table 3: The textures of M_R and the conditions of elements in M_D for getting and breaking the SSA for the classes $\tilde{F}_1$–$\tilde{F}_7$. Here, "All" means the admixture of all three cases A1, A2 and A3.

Class	M_R	Conditions for getting the SSA	Conditions for breaking the SSA
$\tilde{F}_1$	$\begin{pmatrix} x & 0 & 0 \\ 0 & 0 & s \\ 0 & s & 0 \end{pmatrix}$	($\tilde{\text{f}}$1I) $a_2 = b_2 = d_2 = 0$ ($\tilde{\text{f}}$1II)$a_3 = b_3 = d_3 = 0$	$a_2 \neq 0$ (A3), $b_2 \neq 0$ (All), $d_2 \neq 0$ (All) $a_3 \neq 0$ (A3), $b_3 \neq 0$ (All), $d_3 \neq 0$ (All)
$\tilde{F}_2$	$\begin{pmatrix} x & 0 & 0 \\ 0 & t & s \\ 0 & s & 0 \end{pmatrix}$	($\tilde{\text{f}}$2I) $a_2 = b_2 = d_2 = 0$ and $b_1 = d_1 = 0$ ($\tilde{\text{f}}$2II) $a_3 = b_3 = d_3 = 0$	$a_2 \neq 0$ (A3), $b_2 \neq 0$ (All), $d_2 \neq 0$ (All) $b_1 \neq 0$ (All), $d_1 \neq 0$ (All) $a_3 \neq 0$ (A3), $b_3 \neq 0$ (All), $d_3 \neq 0$ (All)
$\tilde{F}_3$	$\begin{pmatrix} x & u & 0 \\ u & t & s \\ 0 & s & 0 \end{pmatrix}$	($\tilde{\text{f}}$3I) $a_1 = b_1 = d_1 = 0$ and $a_2 = b_2 = d_2 = 0$ ($\tilde{\text{f}}$3II) $a_3 = b_3 = d_3 = 0$	$a_1 \neq 0$ (A3), $b_1 \neq 0$ (All), $d_1 \neq 0$ (All) $a_2 \neq 0$ (A3), $b_2 \neq 0$ (All), $d_2 \neq 0$ (All) $a_3 \neq 0$ (A3), $b_3 \neq 0$ (All), $d_3 \neq 0$ (All)
$\tilde{F}_4$	$\begin{pmatrix} x & u & 0 \\ u & 0 & s \\ 0 & s & 0 \end{pmatrix}$	same as $\tilde{F}_3$	same as $\tilde{F}_3$
$\tilde{F}_5$	$\begin{pmatrix} x & u & z \\ u & 0 & s \\ z & s & 0 \end{pmatrix}$	($\tilde{\text{f}}$5I) $a_1 = b_1 = d_1 = 0$ and $a_2 = b_2 = d_2 = 0$ ($\tilde{\text{f}}$5II) $a_2 = b_2 = d_2 = 0$ and $a_3 = b_3 = d_3 = 0$ ($\tilde{\text{f}}$5III) $a_1 = b_1 = d_1 = 0$ and $a_3 = b_3 = d_3 = 0$	$a_1 \neq 0$ (A3), $b_1 \neq 0$ (All), $d_1 \neq 0$ (All) $a_2 \neq 0$ (A3), $b_2 \neq 0$ (All), $d_2 \neq 0$ (All) $a_2 \neq 0$ (A3), $b_2 \neq 0$ (All), $d_2 \neq 0$ (All) $a_3 \neq 0$ (A3), $b_3 \neq 0$ (All), $d_3 \neq 0$ (All) $a_1 \neq 0$ (A3), $b_1 \neq 0$ (All), $d_1 \neq 0$ (All) $a_3 \neq 0$ (A3), $b_3 \neq 0$ (All), $d_3 \neq 0$ (All)
$\tilde{F}_6$	$\begin{pmatrix} x & u & z \\ u & t & s \\ z & s & 0 \end{pmatrix}$	same as $\tilde{F}_5$	same as $\tilde{F}_5$
$\tilde{F}_7$	$\begin{pmatrix} x & 0 & z \\ 0 & t & s \\ z & s & y \end{pmatrix}$	same as $\tilde{F}_5$	same as $\tilde{F}_5$

Table 4: The textures of M_R and the conditions of elements in M_D for getting and breaking the SSA for the classes G_1–G_6 and H_1–H_3. Here, "All" means the admixture of all three cases A1, A2 and A3.

Class	M_R	Conditions for getting the SSA	Conditions for breaking the SSA
G_1	$\begin{pmatrix} 0 & 0 & z \\ 0 & t & 0 \\ z & 0 & 0 \end{pmatrix}$	(g1I) $a_1 = b_1 = d_1 = 0$ (g1II) $a_3 = b_3 = d_3 = 0$	$a_1 \neq 0$ (A3), $b_1 \neq 0$ (All), $d_1 \neq 0$ (All) $a_3 \neq 0$ (A3), $b_3 \neq 0$ (All), $d_3 \neq 0$ (All)
G_2	$\begin{pmatrix} 0 & s & z \\ s & t & 0 \\ z & 0 & 0 \end{pmatrix}$	(g2I) $a_1 = b_1 = d_1 = 0$ and $a_2 = b_2 = d_2 = 0$ (g2II) $a_3 = b_3 = d_3 = 0$	$a_1 \neq 0$ (A3), $b_1 \neq 0$ (All), $d_1 \neq 0$ (All) $a_2 \neq 0$ (A3), $b_2 \neq 0$ (All), $d_2 \neq 0$ (All) $a_3 \neq 0$ (A3), $b_3 \neq 0$ (All), $d_3 \neq 0$ (All)
G_3	$\begin{pmatrix} y & s & z \\ s & t & 0 \\ z & 0 & 0 \end{pmatrix}$	same as G_2	same as G_2
G_4	$\begin{pmatrix} 0 & 0 & z \\ 0 & t & s \\ z & s & 0 \end{pmatrix}$	(g4I) $a_1 = b_1 = d_1 = 0$ (g4II) $a_2 = b_2 = d_2 = 0$ and $a_3 = b_3 = d_3 = 0$	$a_1 \neq 0$ (A3), $b_1 \neq 0$ (All), $d_1 \neq 0$ (All) $a_2 \neq 0$ (A3), $b_2 \neq 0$ (All), $d_2 \neq 0$ (All) $a_3 \neq 0$ (A3), $b_3 \neq 0$ (All), $d_3 \neq 0$ (All)
G_5	$\begin{pmatrix} 0 & 0 & z \\ 0 & t & s \\ z & s & y \end{pmatrix}$	same as G_4	same as G_4
G_6	$\begin{pmatrix} 0 & s & z \\ s & t & u \\ z & u & 0 \end{pmatrix}$	(g6I) $a_1 = b_1 = d_1 = 0$ and $a_2 = b_2 = d_2 = 0$ (g6II) $a_2 = b_2 = d_2 = 0$ and $a_3 = b_3 = d_3 = 0$ (g6III) $a_1 = b_1 = d_1 = 0$ and $a_3 = b_3 = d_3 = 0$	$a_1 \neq 0$ (A3), $b_1 \neq 0$ (All), $d_1 \neq 0$ (All) $a_2 \neq 0$ (A3), $b_2 \neq 0$ (All), $d_2 \neq 0$ (All) $a_2 \neq 0$ (A3), $b_2 \neq 0$ (All), $d_2 \neq 0$ (All) $a_3 \neq 0$ (A3), $b_3 \neq 0$ (All), $d_3 \neq 0$ (All) $a_1 \neq 0$ (A3), $b_1 \neq 0$ (All), $d_1 \neq 0$ (All) $a_3 \neq 0$ (A3), $b_3 \neq 0$ (All), $d_3 \neq 0$ (All)
H_1	$\begin{pmatrix} y & s & 0 \\ s & 0 & u \\ 0 & u & x \end{pmatrix}$	same as G_6	same as G_6
H_2	$\begin{pmatrix} y & s & 0 \\ s & t & u \\ 0 & u & x \end{pmatrix}$	same as G_6	same as G_6
H_3	$\begin{pmatrix} y & s & z \\ s & t & u \\ z & u & x \end{pmatrix}$	same as G_6	same as G_6

First, let us diagonalize the unperturbed part of Eq. (3.6)[2], which satisfies the SSA,

$$
\begin{aligned}
M_\nu^{(0)} M_\nu^{(0)\dagger} &= \begin{pmatrix} A' & B' e^{i\phi'} & B' e^{i\phi'}/c \\ B' e^{-i\phi'} & D' & D'/c \\ B' e^{-i\phi'} c & D'/c & D'/c^2 \end{pmatrix} \\
&= P_\nu^\dagger \begin{pmatrix} A' & B' & B'/c \\ B' & D' & D'/c \\ B'/c & D'/c & D'/c^2 \end{pmatrix} P_\nu \,, \qquad (3.9)
\end{aligned}
$$

with $P_\nu \equiv \mathrm{diag}(e^{i\phi'}, 1, 1)$ and

$$
\begin{aligned}
A' &\equiv A^2 + B^2(1 + 1/c^2)\,, & (3.10)\\
B' e^{i\phi'} &\equiv AB e^{-i\phi} + BD e^{i\phi}(1 + 1/c^2)\,, & (3.11)\\
D' &\equiv B^2 + D^2(1 + 1/c^2)\,. & (3.12)
\end{aligned}
$$

The $M_\nu^{(0)} M_\nu^{(0)\dagger}$ can be diagonalized as $U_\nu^{(0)\dagger} M_\nu^{(0)} M_\nu^{(0)\dagger} U_\nu^{(0)} = \mathrm{diag}((m_1^2)^{(0)}, (m_2^2)^{(0)}, (m_3^2)^{(0)})$, where the mass eigenvalues for Eq. (3.9) are given by

$$
\begin{aligned}
(m_1^2)^{(0)} &= \frac{1}{2}\{D'(1 + 1/c^2) + A' - w\}\,, & (3.13)\\
(m_2^2)^{(0)} &= \frac{1}{2}\{D'(1 + 1/c^2) + A' + w\}\,, & (3.14)\\
(m_3^2)^{(0)} &= 0\,, & (3.15)
\end{aligned}
$$

with $w \equiv \sqrt{4B'^2(1 + 1/c^2) + \{D'(1 + 1/c^2) - A'\}^2}$ and the unitary matrix for diagonalization of Eq. (3.9) is given by

$$
U_\nu^{(0)} = \begin{pmatrix} e^{i\phi'} \cos\theta & e^{i\phi'} \sin\theta & 0 \\ -\frac{c \sin\theta}{\sqrt{1+c^2}} & \frac{c \cos\theta}{\sqrt{1+c^2}} & -\frac{1}{\sqrt{1+c^2}} \\ -\frac{\sin\theta}{\sqrt{1+c^2}} & \frac{\cos\theta}{\sqrt{1+c^2}} & \frac{c}{\sqrt{1+c^2}} \end{pmatrix}\,, \qquad (3.16)
$$

with

$$
\begin{aligned}
\sin\theta &= \sqrt{\frac{-(m_1^2)^{(0)} + A'}{(m_2^2)^{(0)} - (m_1^2)^{(0)}}} = \sqrt{\frac{-(D'(1 + 1/c^2) - A' - w)}{2w}}\,, & (3.17)\\
\cos\theta &= \sqrt{\frac{(m_2^2)^{(0)} - A'}{(m_2^2)^{(0)} - (m_1^2)^{(0)}}} = \sqrt{\frac{D'(1 + 1/c^2) - A' + w}{2w}}\,. & (3.18)
\end{aligned}
$$

As we can see in Eq. (3.16), taking $c = 1$ leads to the exact maximal atmospheric neutrino mixing angle. By adjusting the value of c, we can obtain non-maximal one. In order for $M_\nu^{(0)}$ to be consistent with the experimental data for $\Delta m^2_{\mathrm{sol}}/\Delta m^2_{\mathrm{atm}}$ and

[2] The diagonalization of Eq. (3.2) for the case of $c = 1$ has been discussed in Ref. [18].

$\sin\theta_{12}$, both conditions (i)$D'(1+1/c^2)+A' \gg \omega$ corresponding to $B \gg A, D$ or $A, D \gg B$, and (ii)$4B'^2(1+1/c^2) \gg \{D'(1+1/c^2)-A'\}^2$ corresponding to $A^2 \simeq D^2(1+1/c^2)^2$ should be satisfied.

Next, we examine the effect of the breaking of the SSA. As we have already mentioned in introduction, non-zero m_3 and U_{e3} can be generated by the correction of breaking parameter ϵ. The explicit forms of the mass egenvalues and mixing angles up to the next-leading and leading order approximation of the diagonalization of Eq. (3.9) can be seen in appendix, respectively. Up to the order of ϵ, we can obtain the approximate relations of m_3 and U_{e3} between the cases A1 and A2 as

$$m_3^{(\mathrm{A1})} \simeq m_3^{(\mathrm{A2})}\,, \quad U_{e3}^{(\mathrm{A1})} \simeq U_{e3}^{(\mathrm{A2})}/c^2\,. \tag{3.19}$$

From these relations, we can see that for $c \simeq 1$, the predictions of m_3 and U_{e3} for the case A1 is the almost same as those for the case A2. For $c > 1$, as the deviation of c from 1 becomes larger, the value of U_{e3} in the case A2 becomes larger than that in the case A1 and for $c < 1$ vice versa. On the other hand, the approximate relation of U_{e3} between the cases A3 and A2 (A1) can be obtained as

$$U_{e3}^{(\mathrm{A3})} \simeq \frac{A}{D} U_{e3}^{(\mathrm{A2})} \simeq c^2 \frac{A}{D} U_{e3}^{(\mathrm{A1})}\,, \tag{3.20}$$

from which we can see that the value of U_{e3} in the case A3 is larger than those in the cases A1 and A2 because of the condition (ii). The Jarlskog parameter can be written as [15]

$$J_{CP} = -\frac{\mathrm{Im}[h_{12}h_{23}h_{31}]}{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2}\,, \tag{3.21}$$

with $h = M_\nu M_\nu^\dagger$. Up to the order of ϵ, we can obtain

$$\begin{aligned}
\mathrm{A1:}\quad \mathrm{Im}[h_{12}h_{23}h_{31}] &\simeq -\frac{B'^2D^2}{c^4}|\epsilon|\sin(\phi-\varphi) - \frac{B'D'BD}{c^4}|\epsilon|\sin\varphi\,, && (3.22)\\
\mathrm{A2:}\quad \mathrm{Im}[h_{12}h_{23}h_{31}] &\simeq \frac{B'^2D^2}{c^2}|\epsilon|\sin\phi' + \frac{B'D'BD}{c^2}|\epsilon|\sin(\phi-2\phi')\,, && (3.23)\\
\mathrm{A3:}\quad \mathrm{Im}[h_{12}h_{23}h_{31}] &\simeq \frac{B'^2B^2}{c^2}|\epsilon|\sin\varphi - \frac{B'D'AB}{c^2}|\epsilon|\sin(\phi'+\varphi)\\
&- \frac{B'D'BD}{c^2}|\epsilon|\{\sin(\phi'-\varphi)-\sin(\phi'-\phi-\varphi)\}\,, && (3.24)
\end{aligned}$$

which can lead to non-zero J_{CP} in each case, even if $\varphi = 0$. Note that the form of J_{CP} in the case A2 does not depend on φ. On the other hand, we can see that the second term in Eq. (3.22) and the first term in Eq. (3.24) vanish in the case of $\varphi = 0$. We find that the magnitude of $|J_{CP}|$ can be somewhat enhanced by the existence of non-zero φ in the case A1, as we will see in numerical calculations soon later.

In the next section, we will make the numerical analyses for the neutrino masses, mixing angles, J_{CP} and the effective mass for the neutrinoless double beta decay $\langle m_{ee}\rangle$ in the original case of the SSA and the three cases A1, A2 and A3.

4 The numerical analysis

In this section, we show the numerical results for the original case of the SSA and the three cases A1, A2 and A3. For the original case of the SSA, we can restrict the regions of the input parameters A, B, D, c, ϕ from the experimental data given in Eqs. (1.1) and (1.2) and determine the value of the effective mass for the neutrinoless double beta decay $\langle m_{ee}\rangle$. In addtion to these five parameters, for the cases A1, A2 and A3, we have two more ones $|\epsilon|$ and φ, which allow us to determine the values of U_{e3}, m_3 and J_{CP}.

In Table 5, we have listed the predicted values of $|U_{e3}|$, m_3, J_{CP} and $\langle m_{ee}\rangle$ together with the allowed value of c, for the original case of the SSA and the cases A1, A2 and A3. For the original case of the SSA, we have considered the two cases of $\phi = 0$ and $\phi = 0 \sim 2\pi$. For the cases A1, A2 and A3, the two cases of $\phi = 0 \sim 2\pi, \varphi = 0$ and $\phi, \varphi = 0 \sim 2\pi$ have been considered. In Table 5, we can see that the maximum value of $|U_{e3}|$ in the case A2 is larger than that in the case A1. This is responsible for the deviation of c from 1 in the region of $c > 1$. As seen in Eqs. (3.19) and (3.20), the prediction of m_3 for the case A1 is the almost same as those for the case A2 and the value of $|U_{e3}|$ in the case A3 is larger than those in the cases A1 and A2. As we have described in the previous section, we can also see that the magnitude of $|J_{CP}|$ can be somewhat enhanced by the existence of non-zero φ in the case A1. It is in principle possible to detect $|J_{CP}| \sim \mathcal{O}(10^{-2})$ in the future long-baseline neutrino oscillation experiments. On the other hand, such a enhancement cannot be seen in the cases A2 and A3. Also, the effect of the breaking of the SSA on $\langle m_{ee}\rangle$ cannot be seen.

Because of the texture zeros for M_D in our model, within the framework of the seesaw mechanism, we can expect that the predictions of the low energy observables can be constrained from the baryon asymmetry of the universe through the thermal leptogenesis scenario [19]. In the next section, we will study the the baryon asymmetry of the universe based on the thermal leptogenesis scenario.

5 Thermal leptogenesis

In the thermal leptogenesis scenario [19], a lepton asymmetry is generated by the CP violating out-of-equilibrium decay of heavy right-handed Majorana neutrinos N_i. Recently, it has been pointed out that the charged lepton flavor effects play a crucial role on the dynamics of the thermal leptogenesis below the temperature $T \sim M_1 \sim 10^{12}$ GeV [20]. For 10^9 GeV $\lesssim T \sim M_1 \lesssim 10^{12}$ GeV and for $T \sim M_1 \lesssim 10^9$ GeV, the interactions mediated by the τ and μ are non-negligible. Thus, the baryon asymmetry should be calculated by taking into account the flavor effects. Considering the flavor

Table 5: The predicted values of $|U_{e3}|$, m_3, J_{CP} and $\langle m_{ee}\rangle$ together with the allowed value of c for the original case of the SSA and the cases A1, A2 and A3. Here we take $|\epsilon| = 0 \sim 0.25$.

	SSA ($\phi = 0$)	A1 ($\phi = 0 \sim 2\pi, \varphi = 0$)	A2 ($\phi = 0 \sim 2\pi, \varphi = 0$)	A3 ($\phi = 0 \sim 2\pi, \varphi = 0$)
c	$0.72 \sim 1.4$	$0.66 \sim 1.4$	$0.68 \sim 1.4$	$0.67 \sim 1.4$
$\lvert U_{e3}\rvert$	0	≤ 0.022	≤ 0.026	≤ 0.050
m_3 (eV)	0	$\leq 2.5 \times 10^{-3}$	$\leq 2.6 \times 10^{-3}$	$\leq 2.0 \times 10^{-4}$
J_{CP}	0	$-0.0046 \sim 0.0048$	$-0.0061 \sim 0.0054$	$-0.011 \sim 0.011$
$\langle m_{ee}\rangle$ (eV)	$0.0086 \sim 0.047$	$0.095 \sim 0.052$	$0.010 \sim 0.052$	$0.010 \sim 0.052$
	SSA ($\phi = 0 \sim 2\pi$)	A1 ($\phi, \varphi = 0 \sim 2\pi$)	A2 ($\phi, \varphi = 0 \sim 2\pi$)	A3 ($\phi, \varphi = 0 \sim 2\pi$)
c	$0.74 \sim 1.4$	$0.71 \sim 1.5$	$0.73 \sim 1.5$	$0.72 \sim 1.5$
$\lvert U_{e3}\rvert$	0	≤ 0.031	≤ 0.039	≤ 0.058
m_3 (eV)	0	$\leq 3.1 \times 10^{-3}$	$\leq 3.0 \times 10^{-3}$	$\leq 3.3 \times 10^{-4}$
J_{CP}	0	$-0.0047 \sim 0.0070$	$-0.0049 \sim 0.0067$	$-0.011 \sim 0.011$
$\langle m_{ee}\rangle$ (eV)	$0.011 \sim 0.051$	$0.0095 \sim 0.052$	$0.0099 \sim 0.053$	$0.0099 \sim 0.053$

effects, the CP asymmetry parameter ϵ_i^α is defined as [20]

$$\begin{aligned}
\epsilon_i^\alpha &\equiv \frac{\Gamma(N_i \to HL_\alpha) - \Gamma(N_i \to \bar{H}\bar{L}_\alpha)}{\Gamma(N_i \to HL_\alpha) + \Gamma(N_i \to \bar{H}\bar{L}_\alpha)} \\
&= \frac{1}{8\pi v^2}\frac{1}{(M_D M_D^\dagger)_{ii}}\sum_{j\neq i}\mathrm{Im}[(M_D)_{i\alpha}(M_D^\dagger)_{\alpha j}(M_D M_D^\dagger)_{ij}]f(M_j^2/M_i^2)\,, \qquad (5.1)
\end{aligned}$$

where v is a vacuum expectation value of the electroweak symmetry breaking $v \simeq 174$ GeV and

$$f(x) \equiv \sqrt{x}\left\{1 - (1+x)\ln\frac{1+x}{x} + \frac{1}{1-x}\right\}, \qquad (5.2)$$

with $x \equiv M_j^2/M_i^2$. At the temperature $T \sim M_1 > 10^{12}$ GeV, all the charged leptons are out of equilibrium and the flavor effects are indistinguishable. In this paper, we assume $M_3 > M_2 > M_1 > 10^{12}$ GeV and thus one flavor approximation is valid [3]. In this temperature regime, the CP asymmetry parameter ϵ_i is given by [19, 22]

$$\begin{aligned}
\epsilon_i &\equiv \frac{\Gamma(N_i \to HL) - \Gamma(N_i \to \bar{H}\bar{L})}{\Gamma(N_i \to HL) + \Gamma(N_i \to \bar{H}\bar{L})} \\
&= \frac{1}{8\pi v^2}\frac{1}{(M_D M_D^\dagger)_{ii}}\sum_{j\neq i}\mathrm{Im}[((M_D M_D^\dagger)_{ij}^2]f(M_j^2/M_i^2)\,, \qquad (5.3)
\end{aligned}$$

where $f(x)$ is given in Eq. (5.2). In order to calculate the baryon asymmetry of the universe, we need to solve the Boltzmann equations [23]. Here we use the approximate

[3] In the one flavor approximation, for the hierarchical right-handed Majorana neutrinos, the lower bound on M_1, $M_1 > 4.9 \times 10^8$ GeV, has been known [21].

solution of the Boltzmann equations as [24]

$$\eta_B \simeq 0.0096 \sum_i \epsilon_i \kappa_i\,, \tag{5.4}$$

where η_B is the baryon asymmetry of the universe and κ_i is the so-called dilution factor, which describes the wash-out effect of the generated lepton asymmetry and is approximated as [25]

$$\kappa_i \simeq 0.3\left(\frac{10^{-3}\ \mathrm{eV}}{\tilde{m}_i}\right)\left(\ln\frac{\tilde{m}_i}{10^{-3}\ \mathrm{eV}}\right)^{-0.6}\,,\quad \tilde{m}_i \equiv \frac{(M_D M_D^\dagger)_{ii}}{M_i}\,. \tag{5.5}$$

In this study, we concentrate on the class E with the six realization conditions (e1)–(e6) in Table 1, where M_R is diagonal. Under the six conditions (e1)–(e6), we have listed the forms of M_D, $M_D M_D^\dagger$ and the CP asymmetry parameter ϵ_i in Table 6. Here we denote the class E with the condition (e1) as the case Ee1 and so on. As we can see in Table 6, the forms of M_D, $M_D M_D^\dagger$ and ϵ_i in the cases Ee3 and Ee5 (Ee4 and Ee6) can be obtained from those in the case Ee1 (Ee2) by relabeling the indices of generations as $2 \to 1$ and $2 \to 1, 3 \to 2$, respectively. Thus, the pysical consequances for the CP asymmetry in the cases Eei with $i = 1, 3, 5$ and with $i = 2, 4, 6$ are preserved, respectively. As typical examples, we consider the cases Ee5 and Ee6. In Table 7, we have listed the predicted values of $\sin^2\theta_{23}$, $\sin^2\theta_{12}$, Δm^2_{32}, Δm^2_{21} and $\langle m_{ee}\rangle$ for the cases Ee5 and Ee6. Here we take $M_2 = 5\times 10^{14} \sim 5\times 10^{15}$ GeV and $M_1 = (0.01 \sim 0.1)\times M_2$ for the case Ee5. In order to be consistent with the experimental data, which are given in Eqs. (1.1) and (1.2), and the observed value of η_B, $\eta_B = (5.9 - 6.3)\times 10^{-10}$ [26], we need to take the parameters in M_D as $a_1 \sim \mathcal{O}(1)$, $a_2 \sim \mathcal{O}(0.01)$ and $b_2, d_2 \sim \mathcal{O}(1)$. On the other hand, for the case Ee6, we take $M_2 = 5\times 10^{15} \sim 5\times 10^{16}$ GeV and $M_1 = (0.01 \sim 0.1)\times M_2$. Then, the experimental data force the parameters in M_D to be $a_1 \sim \mathcal{O}(0.01)$, $a_2 \sim \mathcal{O}(10)$ and $b_1, d_1 \sim \mathcal{O}(1)$. Thanks of the texture zeros for M_D, we can obtain the correlation between the low energy observable in the lepton sector and the high energy CP violation necessary for the thermal leptogeneis; the value of $\langle m_{ee}\rangle$ can be constrained from η_B as $0.045\ \mathrm{eV} \lesssim \langle m_{ee}\rangle \lesssim 0.052$ eV in the both cases Ee5 and Ee6.

Finally, we discuss the baryon asymmetry of the universe in the case of the breaking of the SSA. For the case Ee5 with $b_3 \neq 0$ (corresponding to the case A2), we can obtain the forms of M_D and $M_D M_D^\dagger$ as follows:

$$M_D = \begin{pmatrix} a_1 e^{i\xi} & 0 & 0 \\ a_2 & b_2 & d_2 \\ 0 & b_3 e^{i\xi'} & 0 \end{pmatrix} v\,, M_D M_D^\dagger = \begin{pmatrix} a_1^2 & a_1 a_2 e^{i\xi} & 0 \\ a_1 a_2 e^{-i\xi} & a_2^2 + b_2^2 + d_2^2 & b_2 b_3 e^{-i\xi'} \\ 0 & b_2 b_3 e^{i\xi'} & b_3^2 \end{pmatrix} v^2 \tag{5.6}$$

From the above forms, we have found that the correction from the breaking of the SSA does not affect on ϵ_1[4]. For the case Ee6 with $b_3 \neq 0$ (corresponding to the case

[4] This statement holds for the case Ee5 with $d_3 \neq 0$ (corresponding to the case A1), because we can obtain the form of $M_D M_D^\dagger$ by rewriting b_2 and b_3 as d_2 and d_3 in the 2-3 (3-2) and 3-3 elements, respectively.

A2), we have

$$M_D = \begin{pmatrix} a_1 & b_1 & d_1 \\ a_2 e^{i\xi} & 0 & 0 \\ 0 & b_3 e^{i\xi'} & 0 \end{pmatrix} v \,, M_D M_D^\dagger = \begin{pmatrix} a_1^2 + b_1^2 + d_1^2 & a_1 a_2 e^{-i\xi} & b_1 b_3 e^{-i\xi'} \\ a_1 a_2 e^{i\xi} & a_2^2 & 0 \\ b_1 b_3 e^{-i\xi'} & 0 & b_3^2 \end{pmatrix} v^2 \tag{5.7}$$

which leads to the correction for ϵ_1 as

$$\Delta\epsilon_1 = \frac{-1}{8\pi} \frac{b_1^2 b_3^2}{a_1^2 + b_1^2 + d_1^2} \sin(2\xi') f(M_3^2/M_1^2) \,. \tag{5.8}$$

From the constraint of η_B, we have found that the value of b_3 should be of the order of $\mathcal{O}(0.1) - \mathcal{O}(1)$. In Figure. 1, we show the predicted value of η_B as a function of $|\epsilon|$. Here we take the values of three right-handed Majorana masses M_i $(i = 1, 2, 3)$ as $M_2 = 5 \times 10^{15} \sim 5 \times 10^{16}$ GeV, $M_1 = (0.01 \sim 0.1) \times M_2$ and $M_3 = (2 \sim 10) \times M_2$. As we can see in Figure. 1, the order of magnitude of the breaking parameter $|\epsilon|$ in this case is given as $|\epsilon| = (b_3^2/b_1^2) \times (M_1/M_3) \simeq \mathcal{O}(10^{-5}) \sim \mathcal{O}(10^{-3})$[5], which can only generate the very small values of $m_3 \sim \mathcal{O}(10^{-9}) - \mathcal{O}(10^{-6})$, $|U_{e3}| \sim \mathcal{O}(10^{-8}) - \mathcal{O}(10^{-6})$ and $J_{CP} \sim \mathcal{O}(10^{-9}) - \mathcal{O}(10^{-7})$. Thus, observation of $|U_{e3}|$ in the next generation of reactor and long-baseline neutrino experiments will exclude the case Ee6 with $b_3 \neq 0$.

For the case Ee5 with $b_1 \neq 0$ and the case Ee6 with $b_2 \neq 0$ (corresponding to the combination of the cases A2 and A3), we have

$$\begin{aligned} M_D &= \begin{pmatrix} a_1 e^{i\xi} & b_1 e^{i\xi'} & 0 \\ a_2 & b_2 & d_2 \\ 0 & 0 & 0 \end{pmatrix} v \,, \\ M_D M_D^\dagger &= \begin{pmatrix} a_1^2 + b_1^2 & a_1 a_2 e^{i\xi} + b_1 b_2 e^{i\xi'} & 0 \\ a_1 a_2 e^{-i\xi} + b_1 b_2 e^{-i\xi'} & a_2^2 + b_2^2 + d_2^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} v^2 \,, \end{aligned} \tag{5.9}$$

and

$$\begin{aligned} M_D &= \begin{pmatrix} a_1 & b_1 & d_1 \\ a_2 e^{i\xi} & b_2 e^{i\xi'} & 0 \\ 0 & 0 & 0 \end{pmatrix} v \,, \\ M_D M_D^\dagger &= \begin{pmatrix} a_1^2 + b_1^2 + d_1^2 & a_1 a_2 e^{-i\xi} + b_1 b_2 e^{-i\xi'} & 0 \\ a_1 a_2 e^{i\xi} + b_1 b_2 e^{i\xi'} & a_2^2 + b_2^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} v^2 \,, \end{aligned} \tag{5.10}$$

from which we can see that the correction from the breaking of the SSA does not affect on ϵ_1 in the both cases [6] as well as the Ee5 with $b_3 \neq 0$.

---

[5] For the case Ee6 with $d_3 \neq 0$ (corresponding to the case A1), we can obtain the form of $M_D M_D^\dagger$ by rewriting b_1 and b_3 as d_1 and d_3 in the 1-3 (3-1) and 3-3 elements, respectively. Then, we can obtain $\Delta\epsilon_1 = (-1/8\pi)(d_1^2 d_3^2) \sin(2\xi') f(M_3^2/M_1^2)/(a_1^2 + b_1^2 + d_1^2)$. Because of $b_1, d_1 \sim \mathcal{O}(1)$, d_3 should also be of the order of $\mathcal{O}(0.1) - \mathcal{O}(1)$ which leads to $|\epsilon| = (d_3^2/d_1^2) \times (M_1/M_3) \simeq \mathcal{O}(10^{-5}) \sim \mathcal{O}(10^{-3})$.

[6] Similarly to the case Ee5 with $b_3 \neq 0$, this statement holds for the case Ee5 (Ee6) with $b_1 \neq 0$ ($b_2 \neq 0$) corresponding to the combination of the cases A1 and A3, because we can obtain the form of $M_D M_D^\dagger$ by rewriting b_1 and b_2 as d_1 and d_2 in the 1-2 (2-1) and 2-2 elements, respectively.

Table 6: The forms of M_D, $M_D M_D^\dagger$ and the CP asymmetry parameter ϵ_i for the class E with the six conditions (e1)–(e6). Here, the case Ee1 means the class E with the condition (e1) and so on.

Case	M_D/v	$M_D M_D^\dagger/v^2$	ϵ_i
Ee1	$\begin{pmatrix} 0 & 0 & 0 \\ a_2 e^{i\xi} & 0 & 0 \\ a_3 & b_3 & d_3 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & a_2^2 & a_2 a_3 e^{i\xi} \\ 0 & a_2 a_3 e^{-i\xi} & a_3^2+b_3^2+d_3^2 \end{pmatrix}$	$\epsilon_2 = \frac{1}{8\pi} a_3^2 \sin(2\xi) f(M_3^2/M_2^2)$ $\epsilon_3 = \frac{-1}{8\pi} \frac{a_2^2 a_3^2}{a_3^2+b_3^2+d_3^2} \sin(2\xi) f(M_2^2/M_3^2)$
Ee2	$\begin{pmatrix} 0 & 0 & 0 \\ a_2 & b_2 & d_2 \\ a_3 e^{i\xi} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & a_2^2+b_2^2+d_2^2 & a_2 a_3 e^{-i\xi} \\ 0 & a_2 a_3 e^{i\xi} & a_3^2 \end{pmatrix}$	$\epsilon_2 = \frac{-1}{8\pi} \frac{a_2^2 a_3^2}{a_2^2+b_2^2+d_2^2} \sin(2\xi) f(M_3^2/M_2^2)$ $\epsilon_3 = \frac{1}{8\pi} a_2^2 \sin(2\xi) f(M_2^2/M_3^2)$
Ee3	$\begin{pmatrix} a_1 e^{i\xi} & 0 & 0 \\ 0 & 0 & 0 \\ a_3 & b_3 & d_3 \end{pmatrix}$	$\begin{pmatrix} a_1^2 & 0 & a_1 a_3 e^{i\xi} \\ 0 & 0 & 0 \\ a_1 a_3 e^{-i\xi} & 0 & a_3^2+b_3^2+d_3^2 \end{pmatrix}$	$\epsilon_1 = \frac{1}{8\pi} a_3^2 \sin(2\xi) f(M_3^2/M_1^2)$ $\epsilon_3 = \frac{-1}{8\pi} \frac{a_1^2 a_3^2}{a_3^2+b_3^2+d_3^2} \sin(2\xi) f(M_1^2/M_3^2)$
Ee4	$\begin{pmatrix} a_1 & b_1 & d_1 \\ 0 & 0 & 0 \\ a_3 e^{i\xi} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} a_1^2+b_1^2+d_1^2 & 0 & a_1 a_3 e^{-i\xi} \\ 0 & 0 & 0 \\ a_1 a_3 e^{i\xi} & 0 & a_3^2 \end{pmatrix}$	$\epsilon_1 = \frac{-1}{8\pi} \frac{a_1^2 a_3^2}{a_1^2+b_1^2+d_1^2} \sin(2\xi) f(M_3^2/M_1^2)$ $\epsilon_3 = \frac{1}{8\pi} a_1^2 \sin(2\xi) f(M_1^2/M_3^2)$
Ee5	$\begin{pmatrix} a_1 e^{i\xi} & 0 & 0 \\ a_2 & b_2 & d_2 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} a_1^2 & a_1 a_2 e^{i\xi} & 0 \\ a_1 a_2 e^{-i\xi} & a_2^2+b_2^2+d_2^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\epsilon_1 = \frac{1}{8\pi} a_2^2 \sin(2\xi) f(M_2^2/M_1^2)$ $\epsilon_2 = \frac{-1}{8\pi} \frac{a_1^2 a_2^2}{a_2^2+b_2^2+d_2^2} \sin(2\xi) f(M_1^2/M_2^2)$
Ee6	$\begin{pmatrix} a_1 & b_1 & d_1 \\ a_2 e^{i\xi} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} a_1^2+b_1^2+d_1^2 & a_1 a_2 e^{-i\xi} & 0 \\ a_1 a_2 e^{i\xi} & a_2^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\epsilon_1 = \frac{-1}{8\pi} \frac{a_1^2 a_2^2}{a_1^2+b_1^2+d_1^2} \sin(2\xi) f(M_2^2/M_1^2)$ $\epsilon_2 = \frac{1}{8\pi} a_1^2 \sin(2\xi) f(M_1^2/M_2^2)$

Table 7: The allowed values of the parameters in M_D and the predicted values of $\sin^2\theta_{23}$, $\sin^2\theta_{12}$, Δm_{32}^2, Δm_{21}^2 and $\langle m_{ee}\rangle$ in the cases Ee5 and Ee6.

	Ee5	Ee6
a_1	$0.46-0.59$	$0.019-0.023$
a_2	$0.17-0.24$	$6.6-8.8$
b_2	$1.2-1.8$	$1.4-2.0$
d_2	$1.1-1.5$	$1.1-1.9$
ξ	$\sim \pi/2, 3\pi/2$	$\sim \pi/5, 5\pi/6, 5\pi/4, 20\pi/11$
$\sin^2\theta_{23}$	$0.34-0.49$	$0.34-0.49$
$\sin^2\theta_{12}$	$0.26-0.40$	$0.26-0.39$
Δm_{32}^2	$2.2\times10^{-3}-2.8\times10^{-3}$	$2.0\times10^{-3}-2.8\times10^{-3}$
Δm_{21}^2	$7.2\times10^{-5}-8.3\times10^{-5}$	$7.1\times10^{-5}-8.3\times10^{-5}$
$\langle m_{ee}\rangle$	$0.046-0.052$	$0.044-0.052$

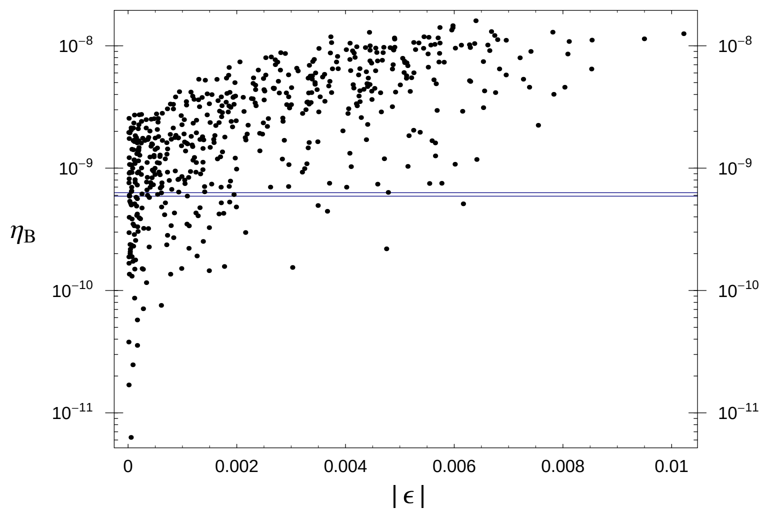

Figure 1: The predicted baryon asymmetry as a function of $|\epsilon|$ in the case Ee6 with $b_3 \neq 0$.

6 Summary

In this study, we have classified the Dirac and the right-handed Majorana neutrino mass matrices which can satisfy the SSA within the framework of the seesaw mechanism, assuming that the Dirac neutrino mass matrix has some texture zeros. We found that the resulting Dirac neutrino mass matrices have rank 2 as well as the rank of the effective neutrino mass matrix, or rank 1, depending only on the textures of M_R^{-1}. We also considered the three cases of breaking the SSA by introducing a complex breaking parameter in the neutrino mass matrix and examined the effects of the breaking of the SSA on $|U_{e3}|, m_3$ and J_{CP}.

We have calculated the baryon asymmetry of the universe in the cases Ee5 and Ee6 which satisfy the SSA in the basis where M_R is diagonal. The implications of the baryon asymmetry for both cases are almost same. We have also discussed the implications of the baryon asymmetry in the case of the breaking of the SSA for the cases Ee5 and Ee6. We have found that only in the case Ee6 with $b_3 \neq 0$ ($d_3 \neq 0$) corresponding to the case A2 (A1), the CP asymmetry parameter ϵ_1 can receive the correction from the breaking of the SSA. From the constraint of the observed value of η_B, the order of magnitude of the breaking parameter ϵ should be of the order of $|\epsilon| \simeq \mathcal{O}(10^{-5}) \sim \mathcal{O}(10^{-3})$, which can only generate the very small values of $m_3 \sim \mathcal{O}(10^{-9}) - \mathcal{O}(10^{-6})$, $|U_{e3}| \sim \mathcal{O}(10^{-8}) - \mathcal{O}(10^{-6})$ and $J_{CP} \sim \mathcal{O}(10^{-9}) - \mathcal{O}(10^{-7})$. Thus, observation of $|U_{e3}|$ in the next generation of reactor and long-baseline neutrino experiments will exclude the case Ee6 with $b_3 \neq 0$ ($d_3 \neq 0$) as well as the original

case of the SSA.

Acknowledgements

I would like to thank Prof. Z.z. Xing for discussion and encouragement in the earlier stage of this work. I would also like to thank W. Chao and H. Zhang for discussions about semi-analytic diagonalization. I would also like to thank L. Shu and K. Matsuda for comments.

A Corrections for mass eigenvalues and eigenstates

In this appendix, we will list the explicite forms of the mass eigenvalues and mixing angles up to the next-leading and the leading order approximation of the diagonalization of Eq. (3.9), respectively. The leading and the next-leading order of corrections for mass eigenvalues $(m_n^2)^{(1)}$'s and $(m_n^2)^{(2)}$'s are given by

$$(m_n^2)^{(1)} = \langle n^{(0)}|\delta\mathcal{M}|n^{(0)}\rangle \quad (n=1,2,3)\,, \tag{A.1}$$

$$(m_n^2)^{(2)} = \sum_{k\neq n}\langle n^{(0)}|\delta\mathcal{M}|k^{(0)}\rangle\langle k^{(0)}|\delta\mathcal{M}|n^{(0)}\rangle \quad (n,k=1,2,3)\,, \tag{A.2}$$

and the leading order of corrections for mass eigenstates $|n^{(1)}\rangle$'s are given by

$$|n^{(1)}\rangle = \sum_{k\neq n}\frac{|k^{(0)}\rangle\langle k^{(0)}|\delta\mathcal{M}|n^{(0)}\rangle}{(m_n^2)^{(0)}-(m_k^2)^{(0)}} \quad (n,k=1,2,3)\,. \tag{A.3}$$

In the next three subsections, we will write down the corresponding expressions for Eqs. (A.1), (A.2) and (A.3) in the three cases A1, A2 and A3.

A.1 The case A1

In the case A1, the correction of the mass eigenvalues for the leading and the next-leading order of the approximation $(m_n^2)^{(1)}$'s and $(m_3^2)^{(2)}$ are given as follows [7]:

$$
\begin{aligned}
(m_1^2)^{(1)} &= 2D^2\left(1+\frac{2}{c^2}\right)\cos(\phi-\varphi)|\epsilon|\sin^2\theta \\
&- 2\frac{BD}{\sqrt{1+\frac{1}{c^2}}}\left(1+\frac{2}{c^2}\right)\cos\varphi|\epsilon|\sin\theta\cos\theta \\
&+ D^2\left(1+\frac{1}{c^2}+\frac{1}{1+c^2}\right)|\epsilon|^2\sin^2\theta\,, \qquad \text{(A.4)} \\
(m_2^2)^{(1)} &= 2D^2\left(1+\frac{2}{c^2}\right)\cos(\phi-\varphi)|\epsilon|\cos^2\theta \\
&+ 2\frac{BD}{\sqrt{1+\frac{1}{c^2}}}\left(1+\frac{2}{c^2}\right)\cos\varphi|\epsilon|\sin\theta\cos\theta \\
&+ D^2\left(1+\frac{1}{c^2}+\frac{1}{1+c^2}\right)|\epsilon|^2\cos^2\theta\,, \qquad \text{(A.5)} \\
(m_3^2)^{(1)} &= \frac{D^2}{c^2(1+c^2)}|\epsilon|^2\,, \qquad \text{(A.6)} \\
(m_3^2)^{(2)} &= \frac{-1}{A'D'(1+c^2)}\left[\frac{B^2D^2D'}{c^4}|\epsilon|^2\right. \\
&+ A'D^4|\epsilon|^2\left(\frac{1}{c^4}+\frac{2\cos(\phi-\varphi)}{c^2(1+c^2)}|\epsilon|+\frac{1}{(1+c^2)^2}|\epsilon|^2\right) \\
&+ \left.2\frac{BD^3B'}{c^2}|\epsilon|^2\left(\frac{\cos\varphi}{c^2}+\frac{\cos(\phi'-\phi+\varphi)}{(1+c^2)}|\epsilon|\right)\right]. \qquad \text{(A.7)}
\end{aligned}
$$

[7] As we can see in Eq. (A.6), the leading order of correction $(m_3^2)^{(1)}$ includes only the term of the order of $|\epsilon|^2$. Thus, we need to take into account the corrections up to the next-leading order for m_3^2, which also includes the terms of the order of $|\epsilon|^2$.

The forms of $\langle k^{(0)}|\delta\mathcal{M}|n^{(0)}\rangle$'s in Eq. (A.3) can be expressed as follows:

$$\begin{aligned}
\langle 3^{(0)}|\delta\mathcal{M}|1^{(0)}\rangle &= \frac{D^2}{c}\left(\frac{e^{-i(\phi-\varphi)}}{c^2}+\frac{1}{1+c^2}|\epsilon|\right)|\epsilon|\sin\theta \\
&- \frac{BD}{c^2\sqrt{1+c^2}}e^{i(\phi'-\phi+\varphi)}|\epsilon|\cos\theta \qquad \text{(A.8)} \\
&= \langle 1^{(0)}|\delta\mathcal{M}|3^{(0)}\rangle^*, \qquad \text{(A.9)} \\
\langle 3^{(0)}|\delta\mathcal{M}|2^{(0)}\rangle &= -\frac{D^2}{c}\left(\frac{e^{-i(\phi-\varphi)}}{c^2}+\frac{1}{1+c^2}|\epsilon|\right)|\epsilon|\cos\theta \\
&- \frac{BD}{c^2\sqrt{1+c^2}}e^{i(\phi'-\phi+\varphi)}|\epsilon|\sin\theta \qquad \text{(A.10)} \\
&= \langle 2^{(0)}|\delta\mathcal{M}|3^{(0)}\rangle^*, \qquad \text{(A.11)} \\
\langle 2^{(0)}|\delta\mathcal{M}|1^{(0)}\rangle &= -2\frac{D^2}{c^2}\left(1+\frac{2}{c^2}\right)\cos(\phi-\varphi)|\epsilon|\sin\theta\cos\theta \\
&+ \frac{BD}{\sqrt{1+\frac{1}{c^2}}}\left(1+\frac{2}{c^2}\right)|\epsilon|(2\cos(\phi'-\phi+\varphi)\cos^2\theta - e^{-i(\phi'-\phi+\varphi)}) \\
&- D^2\left(1+\frac{1}{c^2}\right)|\epsilon|^2\sin\theta\cos\theta \qquad \text{(A.12)} \\
&= \langle 1^{(0)}|\delta\mathcal{M}|2^{(0)}\rangle^*. \qquad \text{(A.13)}
\end{aligned}$$

Using Eqs. (3.17) and (3.18), we can obtain the correction of the three mixing angles for neutrinos as

$$
\begin{aligned}
(U_\nu^{(1)})_{e3} &= -\frac{1}{A'D'\sqrt{1+\frac{1}{c^2}}} \\
&\times \left[\frac{D^2|B'|}{c}e^{i\phi'}\left(\frac{e^{-i(\phi-\varphi)}}{c^2}+\frac{1}{1+c^2}|\epsilon|\right)|\epsilon| - \frac{BDD'}{c^3}e^{i(\phi-\varphi)}|\epsilon|\right], \quad \text{(A.14)} \\
(U_\nu^{(1)})_{\mu 3} &= \frac{1}{A'D'(1+\frac{1}{c^2})^{3/2}} \\
&\times \left[\frac{A'D^2}{c}\left(\frac{e^{-i(\phi-\varphi)}}{c^2}+\frac{1}{1+c^2}|\epsilon|\right)|\epsilon| - \frac{BD|B'|}{c^3}e^{-i(\phi'-\phi+\varphi)}|\epsilon|\right], \quad \text{(A.15)} \\
(U_\nu^{(1)})_{e2} &= \frac{e^{i\phi'}\cos\theta}{w}\left[-2\frac{D^2}{c^2}\left(1+\frac{2}{c^2}\right)\cos(\phi-\varphi)|\epsilon|\sin\theta\cos\theta\right. \\
&+ \frac{BD}{\sqrt{1+\frac{1}{c^2}}}\left(1+\frac{2}{c^2}\right)|\epsilon|(2\cos(\phi'-\phi+\varphi)\cos^2\theta - e^{i(\phi'-\phi+\varphi)}) \\
&- \left. D^2\left(1+\frac{1}{c^2}\right)|\epsilon|^2\sin\theta\cos\theta\right]. \quad \text{(A.16)}
\end{aligned}
$$

A.2 The case A2

In the case A2, the correction of the mass eigenvalues for the leading and the next-leading order of the approximation $(m_n^2)^{(1)}$'s and $(m_3^2)^{(2)}$ are given as follows [8]:

$$
\begin{aligned}
(m_1^2)^{(1)} &= D^2\left(2\cos\varphi + \frac{c^2}{1+c^2}|\epsilon|\right)|\epsilon|\sin^2\theta \\
&- 2\frac{BD}{\sqrt{1+\frac{1}{c^2}}}\cos(\phi' - \phi + \varphi)|\epsilon|\sin\theta\cos\theta\,, \qquad \text{(A.17)} \\
(m_2^2)^{(1)} &= D^2\left(2\cos\varphi + \frac{c^2}{1+c^2}|\epsilon|\right)|\epsilon|\cos^2\theta \\
&+ 2\frac{BD}{\sqrt{1+\frac{1}{c^2}}}\cos(\phi' - \phi + \varphi)|\epsilon|\sin\theta\cos\theta\,, \qquad \text{(A.18)} \\
(m_3^2)^{(1)} &= \frac{D^2}{1+c^2}|\epsilon|^2\,, \qquad \text{(A.19)} \\
(m_3^2)^{(2)} &= \frac{-1}{A'D'(1+\frac{1}{c^2})}\left[\frac{B^2D^2D'}{c^2}|\epsilon|^2 + A'D^4|\epsilon|^2\left(\frac{1}{c^2} + \frac{2\cos\varphi}{1+c^2}|\epsilon| + \frac{c^2}{(1+c^2)^2}|\epsilon|^2\right)\right. \\
&\left. - 2\frac{BD^3B'}{c^2}|\epsilon|^2\left(\cos(\phi'+\varphi) + \frac{c^2}{(1+c^2)^2}\cos(\phi'-\phi+\varphi)|\epsilon|\right)\right]. \qquad \text{(A.20)}
\end{aligned}
$$

[8] Similarly to the case A1, the leading order of correction $(m_3^2)^{(1)}$ includes only the term of the order of $|\epsilon|^2$ in Eq. (A.19). Thus, we need to take into account the corrections up to the next-leading order for m_3^2, which also includes the terms of the order of $|\epsilon|^2$.

The forms of $\langle k^{(0)}|\delta\mathcal{M}|n^{(0)}\rangle$'s in Eq. (A.3) can be expressed as follows:

$$
\begin{aligned}
\langle 3^{(0)}|\delta\mathcal{M}|1^{(0)}\rangle &= \frac{D^2}{c}\left(e^{i\varphi}+\frac{c^2}{1+c^2}|\epsilon|\right)|\epsilon|\sin\theta \\
&- \frac{BD}{\sqrt{1+c^2}}e^{i(\phi'-\phi+\varphi)}|\epsilon|\cos\theta && \text{(A.21)} \\
&= \langle 1^{(0)}|\delta\mathcal{M}|3^{(0)}\rangle^*\,, && \text{(A.22)} \\
\langle 3^{(0)}|\delta\mathcal{M}|2^{(0)}\rangle &= -\frac{D^2}{c}\left(e^{i\varphi}+\frac{c^2}{1+c^2}|\epsilon|\right)|\epsilon|\cos\theta \\
&- \frac{BD}{\sqrt{1+c^2}}e^{i(\phi'-\phi+\varphi)}|\epsilon|\sin\theta && \text{(A.23)} \\
&= \langle 2^{(0)}|\delta\mathcal{M}|3^{(0)}\rangle^*\,, && \text{(A.24)} \\
\langle 2^{(0)}|\delta\mathcal{M}|1^{(0)}\rangle &= -D^2\left(2\cos\varphi+\frac{c^2}{1+c^2}|\epsilon|\right)|\epsilon|\sin\theta\cos\theta \\
&+ \frac{BD}{\sqrt{1+\frac{1}{c^2}}}|\epsilon|(2\cos(\phi'-\phi+\varphi)\cos^2\theta-e^{-i(\phi'-\phi+\varphi)}) && \text{(A.25)} \\
&= \langle 1^{(0)}|\delta\mathcal{M}|2^{(0)}\rangle^*\,. && \text{(A.26)}
\end{aligned}
$$

Using Eqs. (3.17) and (3.18), we can obtain the correction of the three mixing angles for neutrinos as

$$
\begin{aligned}
(U_\nu^{(1)})_{e3} &= \frac{-1}{A'D'\sqrt{1+\frac{1}{c^2}}} \\
&\times \left[\frac{D^2|B'|}{c}e^{i\phi'}\left(e^{-i\varphi}+\frac{c^2}{1+c^2}|\epsilon|\right)|\epsilon| - \frac{BDD'}{c}e^{i(\phi-\varphi)}|\epsilon|\right], && \text{(A.27)} \\
(U_\nu^{(1)})_{\mu 3} &= \frac{1}{A'D'(1+\frac{1}{c^2})^{3/2}} \\
&\times \left[\frac{A'D^2}{c}\left(e^{-i\varphi}+\frac{c^2}{1+c^2}|\epsilon|\right)|\epsilon| - \frac{BD|B'|}{c}e^{-i(\phi'-\phi+\varphi)}|\epsilon|\right], && \text{(A.28)} \\
(U_\nu^{(1)})_{e2} &= \frac{e^{i\phi'}\cos\theta}{w}\left[-D^2\left(2\cos\varphi+\frac{c^2}{1+c^2}|\epsilon|\right)|\epsilon|\sin\theta\cos\theta\right. \\
&+ \left.\frac{BD}{\sqrt{1+\frac{1}{c^2}}}|\epsilon|(2\cos(\phi'-\phi+\varphi)\cos^2\theta-e^{i(\phi'-\phi+\varphi)})\right]. && \text{(A.29)}
\end{aligned}
$$

A.3 The case A3

In the case A3, the correction of the mass eigenvalues for the leading and the next-leading order of the approximation $(m_n^2)^{(1)}$'s and $(m_3^2)^{(2)}$ are given as follows [9]:

$$
\begin{aligned}
(m_1^2)^{(1)} &= 2B^2|\epsilon|\cos^2\theta + 2B^2|\epsilon|\cos\varphi\sin^2\theta \\
&- 2BD\sqrt{1+\frac{1}{c^2}}\cos(\phi'-\phi'')|\epsilon|\sin\theta\cos\theta \\
&- 2\frac{AB}{\sqrt{1+\frac{1}{c^2}}}\cos(\phi'+\phi'')|\epsilon|\sin\theta\cos\theta \\
&+ B^2\frac{c^2}{1+c^2}|\epsilon|^2\sin^2\theta + B^2|\epsilon|^2\cos^2\theta\,, \qquad \text{(A.30)} \\
(m_2^2)^{(1)} &= 2B^2|\epsilon|\sin^2\theta + 2B^2|\epsilon|\cos\varphi\cos^2\theta \\
&+ 2BD\sqrt{1+\frac{1}{c^2}}\cos(\phi'-\phi'')|\epsilon|\sin\theta\cos\theta \\
&+ 2\frac{AB}{\sqrt{1+\frac{1}{c^2}}}\cos(\phi'+\phi'')|\epsilon|\sin\theta\cos\theta \\
&+ B^2\frac{c^2}{1+c^2}|\epsilon|^2\cos^2\theta + B^2|\epsilon|^2\sin^2\theta\,, \qquad \text{(A.31)} \\
(m_3^2)^{(1)} &= \frac{B^2}{1+c^2}|\epsilon|^2\,, \qquad \text{(A.32)} \\
(m_3^2)^{(2)} &= \frac{-1}{A'D'(1+\frac{1}{c^2})}\left[\frac{A^2B^2D'}{c^2}|\epsilon|^2 + A'B^4|\epsilon|^2\left(\frac{1}{c^2}+\frac{2\cos\varphi}{1+c^2}|\epsilon| + \frac{c^2}{(1+c^2)^2}|\epsilon|^2\right)\right. \\
&\left. - 2\frac{AB^3B'}{c^2}|\epsilon|^2\left(\cos(\phi'+\varphi) + \frac{c^2}{1+c^2}\cos(\phi'-\phi+\varphi)|\epsilon|\right)\right]\,, \qquad \text{(A.33)}
\end{aligned}
$$

where $\phi'' = \phi + \varphi$.

[9] Similarly to the cases A1 and A2, the leading order of correction $(m_3^2)^{(1)}$ given in Eq. (A.32) includes only the term of the order of $|\epsilon|^2$. Thus, we need to take into account the next-leading order of corrections, which also includes the terms of the order of $|\epsilon|^2$. However, the terms of the order of $|\epsilon|^2$ in the leading and the next-leading order corrections almost cancel. Therefore, we need to take into account the higer order corrections for m_3^2 in the case A3. Here we do not write down the expressions. We checked the results for m_3^2 up to the forth order, comparing with the numerical calculation of the diagonalization and found that this perturbation is not good for m_3^2 in the case A3.

The forms of $\langle k^{(0)}|\delta\mathcal{M}|n^{(0)}\rangle$'s in Eq. (A.3) can be expressed as follows:

$$\begin{aligned}
\langle 3^{(0)}|\delta\mathcal{M}|1^{(0)}\rangle &= \frac{B^2}{c}\left(e^{i\varphi}+\frac{c^2}{1+c^2}|\epsilon|\right)|\epsilon|\sin\theta \\
&- \frac{AB}{\sqrt{1+c^2}}e^{i(\phi'+\phi'')}|\epsilon|\cos\theta && \text{(A.34)} \\
&= \langle 1^{(0)}|\delta\mathcal{M}|3^{(0)}\rangle^*\,, && \text{(A.35)} \\
\langle 3^{(0)}|\delta\mathcal{M}|2^{(0)}\rangle &= -\frac{B^2}{c}\left(e^{i\varphi}+\frac{c^2}{1+c^2}|\epsilon|\right)|\epsilon|\cos\theta \\
&- \frac{AB}{\sqrt{1+c^2}}e^{i(\phi'+\phi'')}|\epsilon|\sin\theta && \text{(A.36)} \\
&= \langle 2^{(0)}|\delta\mathcal{M}|3^{(0)}\rangle^*\,, && \text{(A.37)} \\
\langle 2^{(0)}|\delta\mathcal{M}|1^{(0)}\rangle &= 2B^2(1-\cos\varphi)|\epsilon|\sin\theta\cos\theta + B^2\frac{1}{1+c^2}|\epsilon|^2\sin\theta\cos\theta \\
&+ BD\sqrt{1+\frac{1}{c^2}}|\epsilon|(2\cos(\phi'-\phi'')\cos^2\theta - e^{-i(\phi'-\phi'')}) \\
&+ \frac{AB}{\sqrt{1+\frac{1}{c^2}}}|\epsilon|(2\cos(\phi'+\phi'')\cos^2\theta - e^{-i(\phi'+\phi'')}) && \text{(A.38)} \\
&= \langle 1^{(0)}|\delta\mathcal{M}|2^{(0)}\rangle^*\,. && \text{(A.39)}
\end{aligned}$$

Using Eqs. (3.17) and (3.18), we can obtain the correction of the three mixing angles

for neutrinos as

$$
\begin{aligned}
(U_\nu^{(1)})_{e3} &= \frac{1}{A'D'\sqrt{1+\frac{1}{c^2}}} \\
&\times \left[-\frac{B^2|B'|}{c}e^{i\phi'}\left(e^{-i\varphi}+\frac{c^2}{1+c^2}|\epsilon|\right)|\epsilon|+\frac{ABD'}{c}e^{-i\phi''}|\epsilon|\right], \qquad \text{(A.40)} \\
(U_\nu^{(1)})_{\mu 3} &= \frac{1}{A'D'(1+\frac{1}{c^2})^{3/2}} \\
&\times \left[\frac{A'B^2}{c}\left(e^{-i\varphi}+\frac{c^2}{1+c^2}|\epsilon|\right)|\epsilon|-\frac{AB|B'|}{c}e^{-i(\phi'+\phi'')}|\epsilon|\right], \qquad \text{(A.41)} \\
(U_\nu^{(1)})_{e2} &= \frac{e^{i\phi'}\cos\theta}{w}\left[2B^2(1-\cos\varphi)|\epsilon|\sin\theta\cos\theta+B^2\frac{1}{1+c^2}|\epsilon|^2\sin\theta\cos\theta\right. \\
&+ BD\sqrt{1+\frac{1}{c^2}}|\epsilon|(2\cos(\phi'-\phi'')\cos^2\theta-e^{i(\phi'-\phi'')}) \\
&+ \left.\frac{AB}{\sqrt{1+\frac{1}{c^2}}}|\epsilon|(2\cos(\phi'+\phi'')\cos^2\theta-e^{i(\phi'+\phi'')})\right]. \qquad \text{(A.42)}
\end{aligned}
$$

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